

**Problem 1.** (15 points)

To help his teammates finish up an activity after class, Jorge agrees to find the mass (in grams) of the last four objects himself. The four sets of measurements are shown below.

- a. Known value: 32.0  
Measured values: 29.7, 33.9, 32.1, 32.3
- b. Known value: 9.8  
Measured values: 9.8, 7.2, 13.8, 8.4
- c. Known value: 158  
Measured values: 102, 176, 201, 84
- d. Known value: 0.43  
Measured values: 0.20, 0.21, 0.20, 0.19

A. For each set, indicate if the measurements are accurate, precise, both or neither and explain why.

a.) These measurements are both accurate and precise. The average of all 4 measurements is 32.0 which is the same as the known value, which makes the measurements accurate. The measurements are also precise because they are all pretty close.

b.) These measurements are neither accurate nor precise. The average of all 4 measurements is 9.8 which is equal to the known value of 9.8, making the measurements NOT accurate. They are also not precise b/c the msts are not close at all. They are very spread out.

c.) These measurements are accurate but not precise. The average of all 4 measurements is 141.75 which is nowhere close to the known value of 158, making the measurements NOT accurate. They are also not precise b/c the msts are not close. They are very spread out.

d.) These measurements are not accurate, but are precise. The average of all 4 measurements is 0.20 which is nowhere near the known value of 0.43, making the msts NOT accurate. They are precise however b/c the msts are close together.

B. For each object, what mass measurement should Jorge report to his teammates and what is the uncertainty in his measurements?

For each object Jorge should report the average mass. So for example

A → 32.0    B → 9.8    C → 141    D → 0.20

To find the uncertainty of his measurements he needs to find the difference between the highest value and average, and between the lowest value and average for each object. Then take the larger number of the two and that will be Jorge's uncertainty for each object.

A.  $33.9 - 32.0 = 1.9$     B.  $13.8 - 9.8 = 4.0$     C.  $201 - 141 = 60$     D.  $0.21 - 0.20 = 0.01$   
 $32.0 - 29.7 = 2.3$      $9.8 - 7.2 = 2.6$      $141 - 84 = 57$      $0.20 - 0.19 = 0.01$   
either one

Now the reason I chose to take the average number of the 4 measurements & find the uncertainty and not the known number is because that is not what we were taught in class.

See comments on next page

## Problem 1 (cont.)

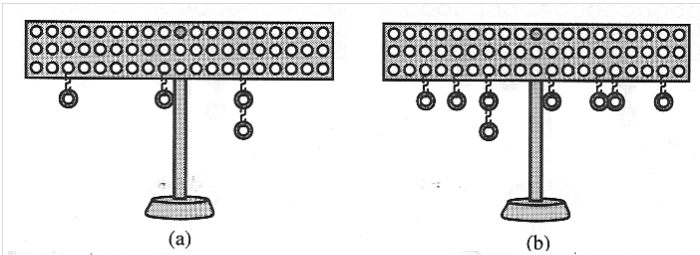
Dr. Saul's comments on Problem 1: *The main difficulties I saw with this problem were the following:*

- *Confusing accuracy and precision*
- *Just using the difference between the largest number and the average rather than looking for the largest difference between a measurement and the average. (Alternatively, you could have used half the difference between the largest and smallest measurements as the uncertainty.)*
- *Significant digits – your answers should have the same precision as the numbers used to calculate them. Thus the average for the data set in part c is 141, not 140.75.*

*The reason you use the average of your measurements and not the known value is that the uncertainty should be determined from your measurements, that is, it's a measure of how precise your measurements are. For example, if your uncertainty is approximately 1/3 of your average value or more, the measurements are not very precise. This is because the spread of values is almost equal to or more than the average value. And if the known value is not within your uncertainty of your average measured value, this is an indication that your measured average value is not accurate.*

**Problem 2** (10 points)

For each of the balance situations below, indicate what you could do so that the two sides will balance. Explain your reasoning. If nothing needs to be done, say that and explain why.



You can use the turning effect to find how to balance each scale. Take the turning effect (mass times length from fulcrum) for each side to determine this. If multiple bolts on each side, find turning effect for each one, then add together to find the effect for that side.

A.   
 Left side: Bolt 1, Bolt 2   
 $7 \times 1 = 7$   $1 \times 7 = 7$    
 $7 + 7 = 14$    
 Right side: Bolt (3+4)   
 $4 \times 2 = 8$    
 $14 \neq 8$

In this case, the 2 sides are equal & nothing needs to be done to balance the scale, it is already balanced..

B. Left Side   
 Bolt 1 Bolt 2 Bolt (3+4)   
 $7 \times 1 = 7$   $5 \times 1 = 5$   $3 \times 2 = 6$    
 $7 + 5 + 6 = 18$

Right Side   
 Bolt 5 Bolt 6 Bolt 7 Bolt 8   
 $1 \times 1 = 1$   $4 \times 1 = 4$   $5 \times 1 = 5$   $8 \times 1 = 8$    
 $1 + 4 + 5 + 8 = 18$

Once again, this scale is already balanced & nothing needs to be

Dr. Saul's comments: This is a pretty good solution with a definition of the turning effect that includes add the effect of all the washers on a given side. So in both cases above, the balance is balanced since the turning effect on the right hand side equals the turning effect on the left hand side. Another solution is shown on the next page naming the turning affect and giving the formula. This solution is also pretty good but could be improved if the equation summed the  $M \times L$ 's on each side of the balance.

## Problem 2 (cont.)

For balance (a) nothing needs to be done to make the two sides balance because the turning effect is equal on each side.

$$M_1 L_1 = M_2 L_2$$

$$1(6) + 1(1) = 2(4)$$

$$8 = 8$$

For balance (b) nothing needs to be done to make the two sides balance because the turning effect is equal on each side.

$$M_1 L_1 = M_2 L_2$$

$$1(7) + 1(5) + 2(3) = 1(1) + 1(4) + 1(5) + 1(8)$$

$$18 = 18$$

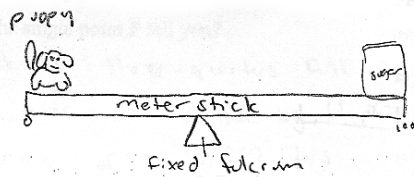
**Problem 3** (10 points)

You would like to determine the mass of your new puppy, but you don't have a scale or a balance. While poking around the pantry for a snack you find a 1 kg bag of sugar and a meter stick. Explain carefully how you could use these objects to determine the mass of your puppy.

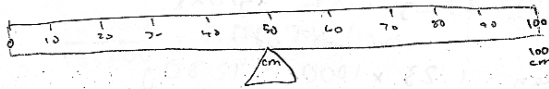
Dr. Saul's comments: Key points

- Use the meter stick and a fulcrum to make a balance
- Placing the puppy on one side of the balance and the bag of sugar on the other, need to adjust puppy and sugar so that system is balanced and turning affects on both sides are equal.
- Use  $M \times L$  (right side) =  $m \times l$  (left side) to find mass of puppy from mass of sugar and distances of the sugar and the puppy from the fulcrum.

Student Solution 1:



By using  $M_1 L_1 = M_2 L_2$  we can determine the mass of the puppy. A meter stick is broken into equal sections so this is a standard measurement device.



- Place the fulcrum directly at 50 cm point or directly in the middle of the meter stick. The two sides should balance before starting the experiment.
- Place bag of sugar on right side and puppy on left side at the same distance from the fulcrum (touching both ends is a good place to start)
- Whichever side touches the floor has a greater turning effect so the other side must compensate.

Using  $M_1 L_1 = M_2 L_2$  we can determine mass.

- If puppy and bag were equal at this point then the puppy's mass would be:  $(1 \text{ kg}) (50 \text{ cm}) = 50 \text{ kg}$ .

if the bag had a greater turning effect in the beginning then the bag must be moved closer to the fulcrum.

Find a point where the bag and puppy balance. Use this point (cm)  $\times$  1 kg and mass is found.

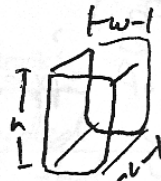
## Problem 3 (continued)

The solution on the previous page is pretty good. It definitely hits the key points. However it is hard to follow because it jumps around a bit. In addition, putting the puppy and sugar at the ends of the meter stick may not be the best way to start. A better way might be to place both the puppy and bag of sugar halfway between the fulcrum and the ends. That way we have room to adjust the turning affect of the bag of sugar regardless of which side has the greater turning affect. Explanation of how to use the equation could be clearer.

While I was thinking in terms of a balance, this is not the only way to answer this question. Below is a creative solution using density and volume, instead of a balance. See the Problem 4 solution on how to improve the argument for using density to find mass.

Assuming that your puppy has the same density of a bag of sugar you can ...

- ① Find the exact dimensions of the 1 kg bag of sugar using the meter stick. You will need to find the height  $\times$  width  $\times$  length!



- ② Next, you will need to find the exact dimensions of your puppy. To do this, you will have to measure your dog in several different parts (i.e. legs, body, head, etc.) but ALWAYS find dimensions by multiplying height  $\times$  width  $\times$  length.
- ③ Once you have the <sup>cube</sup> dimensions of both your puppy and the bag of sugar, divide the "volume" (~~the~~ cubic dimensions of your dog) by the volume of the 1 kg bag of sugar.
- ④ Multiply this quotient by 1 (which represents the mass of the 1 kg bag of sugar), and you will have an estimate of how much your puppy weighs.

**Problem 4** (10 points)

One cubic centimeter of copper has a mass of 8.9 g.

What is the volume of a piece of copper with a mass of 1.23 kg? Explain your reasoning

For full credit, do not use algebra.

*Dr. Saul's comments: The key to this problem is not finding the answer but understanding the process to get the answer. Here we find the volume of the piece with a mass of 1.23 kg by finding out how many 8.9 g pieces of copper it takes to make 1.23 kg and then multiplying by 1 cm<sup>3</sup> per piece. Make sure that unit conversions like going from kg to g are explicit, i.e. show them. One way to do this is like this:*

*1.23 kg x 1000 g / kg = 1230 g (if you clearly showed how you went from kg to g and lost points, come show me to get those points back.)*

Here are some of the better student solutions to this problem:

*Student Solution 1*

*Given: 8.9g of copper has 1cm<sup>3</sup> of volume*

*Procedure: convert 1.23 kg to 1230g*

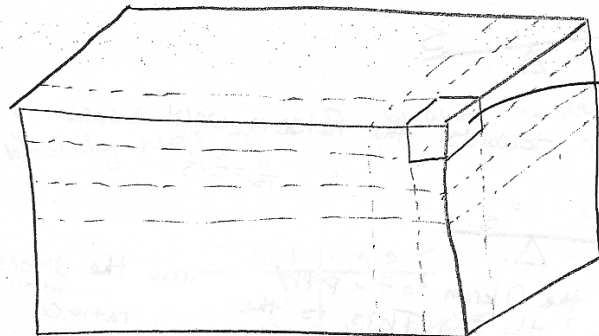
*Divide  $\frac{1230g}{8.9g}$  to find how many cm<sup>3</sup> fit into the number of copper*

*Answer: 138.2cm<sup>3</sup> = volume of a piece of copper with a mass of 1.23kg.*

Handwritten calculations for Student Solution 1:

$$1000g = 1kg$$

$$1.23kg = 1230g$$

$$\begin{array}{r} 1.23 \\ \times 1000 \\ \hline 1230g \end{array}$$


*1 cm<sup>3</sup> = 8.9 g*

$$\frac{1230g}{8.9g} = 138.2$$

$$1230g = 138.2 cm^3$$

*This is a pretty good solution but it could be improved if the reasoning behind the division step was clearer. (Technically if you want to do the calculation in one step you would have to divide 1,230 g by 8.9 g/cm<sup>3</sup>.) Also, since the density only has 2 significant figures, your answer should also only have 2 significant figures. So V = 140 cm<sup>3</sup>*

## Problem 4 (cont.)

## Student Solution 2

$1 \text{ cm}^3 = 8.9 \text{ g}$

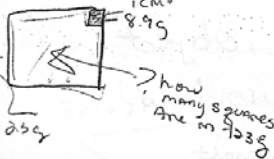
$\begin{array}{r} 100 \\ \times 1.23 \\ \hline 1230 \end{array}$ 
 to find 1.23 kg in grams

$123 / 8.9 = 13.820$  to find how many  $1 \text{ cm}^3 / 8.9 \text{ g}$  pieces are in 123 grams of copper

Therefore, by dividing 123 g by 8.9 g you obtain 13.820 which is the volume b/c for every 8.9 g there is  $1 \text{ cm}^3$

$13.820 \times 1 \text{ cm}^3 = 13.820 \text{ cm}^3$

Volume =  $13.820 \text{ cm}^3$



Note that the reasoning in solution 2 above and solution 3 below in the division step is clearer.

## Student solution 3:

A piece of copper with a mass of 1.23 kg is the same as 1,230 g b/c there are a thousand grams for every one kilo.

For every one cubic centimeter of copper there is a mass of 8.9 g. To find the volume of the 1,230 g piece of copper, I need to find out how many times 8.9 divides into 1,230, which is 138.2. Since  $1,230 \div 8.9 = 138.2$  this gives us the volume of piece of copper. So, for every  $1 \text{ cm}^3 = 8.9 \text{ g}$  there is  $138.2 \text{ cm}^3 = 1.23 \text{ kg}$



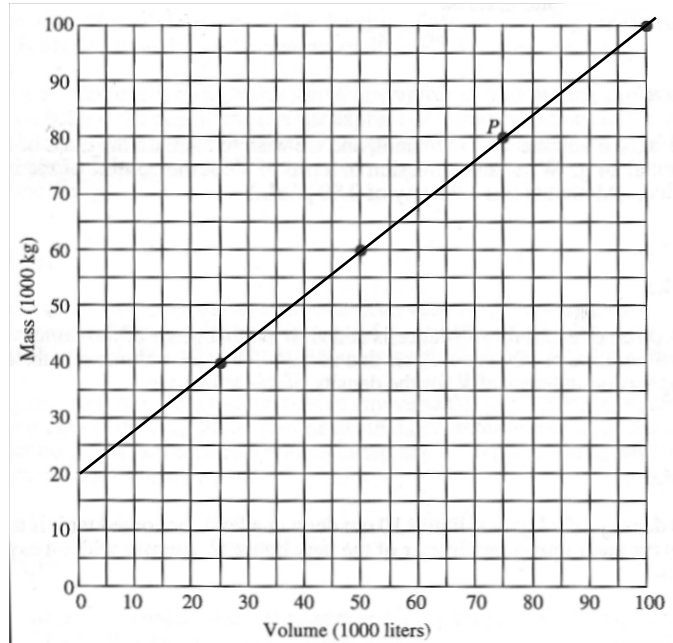
**Problem 5** (20 points)

A tank truck is used to haul a certain liquid. The truck has different masses depending on how full the tank is. The trucker is more concerned with the total mass of the truck and load than the mass of the liquid alone. Below is a graph of the mass of the loaded truck versus volume of liquid in the tank. The points represent the tank being one-quarter full, half-full, three-quarters full, and full.

a. What does the single point P tell you?

*Student Solution:*

P is the pt where the truck is at three quarters full. This means the total mass of truck is at 80,000 kg with a liquid of a volume of 75,000L



*Dr. Saul:* To receive full credit on part (a.), you needed to indicate that the coordinates of point P told you the mass of the truck + the liquid and the volume of the liquid in the truck and provide the values with units..

b. What is the density of the liquid when the truck is three-quarters full? Explain how you can tell.

There were two ways to find the answer to part b. One is realize that the Density of an object = the mass of the object / the volume of the object. Using point P,

$$D = \frac{\text{Mass of Liquid}}{\text{Volume of Liquid}} = \frac{M_{\text{truck+liquid}} - M_{\text{truck}}}{V_{\text{liquid}}}$$

$$\frac{80,000\text{kg} - 20,000\text{kg}}{75,000\text{l}} = \frac{60,000\text{kg}}{75,000\text{l}} = 0.80\text{kg / l}$$

(See part c on how to determine the mass of just the truck.)

*Student Solution:*

density =  $m/v$  The Mass of the truck is 20 (1000 kg)  
 The Mass of the truck & 3/4 tank is 80 (1000kg). So the liquid itself has a mass of 60 (1000 kg)  
 → Mass = 60 (1000 kg) / Volume = 75 (1000 liters)

Thus

$$\frac{60}{75} = \text{density}$$

$$0.8 \text{ kg/liter}$$

This is a good solution except for the tendency to leave the units off of numbers and not label the final answer.

The other way to do this problem is to realize after the initial point with volume = 0, the increase in mass in entirely due to the liquid. Thus the slope of the Mass vs. Volume line shown above is the density of the liquid. Picking the points where the truck is one quarter full of liquid and where the truck is full of liquid:

$$\text{slope} = \frac{\text{rise}}{\text{run}} = \frac{\Delta \text{Mass}}{\Delta \text{Volume}} = \frac{M_{\text{full}} - M_{\text{one quarter full}}}{V_{\text{full}} - V_{\text{one quarter full}}} = \frac{100,000\text{kg} - 40,000\text{kg}}{100,000\text{l} - 25,000\text{l}} = \frac{60,000\text{kg}}{75,000\text{l}} = 0.80\text{kg / l}$$

$$\text{density of liquid} = \text{slope} = 0.80 \text{ kg / l}$$

## Problem 5 (cont.)

c. What is the mass of the empty truck? Explain how you found your answer.

Again, there are two ways to answer this question.

Student solution 1:

Thus, a full tank (4 quarters) of liquid has a mass of 80,000 kg.

Each quarter tank is 20 ~~kg~~ (1000 kg)  $\xrightarrow{\uparrow}$   
 The total Mass w/a full tank is 100 (1000 kg)  
 Take away the 80 (1000 kg) of liquid & you are left  
 with 20 (1000 kg) which is the mass of the truck.

Student Solution 2: The line refers to the trendline through the points.

20,000 kg: continue the line until it reaches the y-axis. This  
 # is the weight of the empty truck b/c the the volume is equal  
 to zero where the line touches the y axis

The point where the trend line crosses the vertical (y) axis is the y-intercept. This is the point where the truck is empty and volume and mass of the liquid is zero.

**Problem 6** (15 points)

An instructor gives a painted piece of metal to 2 students and asks: "This is one of the metals listed in the table of densities. What metal do you think it is?"

The students measure the object and find that the mass is  $139.2 \pm 0.1 \text{ g}$  and the volume is  $16.0 \pm 0.5 \text{ cm}^3$ .

Student 1 says: "It must be nickel."

Student 2 says: "Don't forget the uncertainty. It might be silver."

Table of densities

Substance	Density
Tungsten	19 g/cm <sup>3</sup>
Lead	11.3 g/cm <sup>3</sup>
Silver	10.5 g/cm <sup>3</sup>
Copper	8.9 g/cm <sup>3</sup>
Nickel	8.7 g/cm <sup>3</sup>
Brass	8.5 g/cm <sup>3</sup>
Iron	7.9 g/cm <sup>3</sup>

A. What would you conclude from the data?

I would conclude that with such a large uncertainty in the volume, it could be either Brass, Nickel, or Copper. They all fit within the range of uncertainty.

Dr. Saul: Good solution, but should also mention that is the sample is probably not silver since the density for silver lies outside the range of possible densities. This is a composite solution from 2 students.

Handwritten student work showing mass and volume uncertainty ranges, density calculations for various mass/volume combinations, and a conclusion that the metal could be Brass, Nickel, or Copper.

Mass:  $139.1 \text{ g}$  to  $139.3 \text{ g}$  (range  $\pm 0.1 \text{ g}$ )

Volume:  $15.5 \text{ cm}^3$  to  $16.5 \text{ cm}^3$  (range  $\pm 0.5 \text{ cm}^3$ )

Possibilities of density:

- $\frac{139.1 \text{ g}}{15.5 \text{ cm}^3} \approx 8.97 \frac{\text{g}}{\text{cm}^3}$  (highest)
- $\frac{139.3 \text{ g}}{16.5 \text{ cm}^3} \approx 8.44 \frac{\text{g}}{\text{cm}^3}$  (lowest)

It could be Brass, Nickel, or Copper.

B. Do you agree with student 1, student 2, or neither? Explain your reasoning.

Student Solution: (Dr. Saul: hits all the key points)

disagree with student 1 b/c it doesn't have to be nickel. (like in part a, it could be Brass, Nickel, or copper.)

I disagree with Student 2 b/c it cannot be silver even when remembering the uncertainty.

But I do think it is imperative to remember the uncertainty so I do agree with that part of the statement. 😊