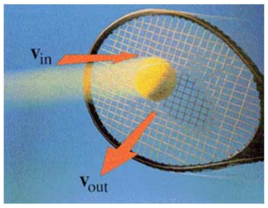
Chapter 7

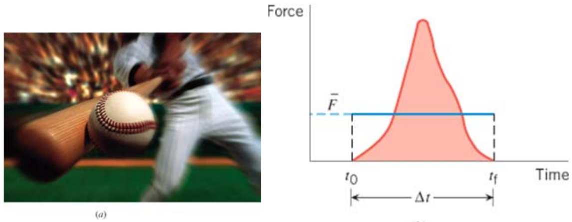
Impulse and Momentum



Goals for Chapter 7

- To study **impulse** and **momentum**.
- To understand **conservation of momentum**.
- To study momentum changes during **collisions**.
- To understand **center of mass** and how forces act on the c.o.m.
- To apply momentum to rocket propulsion.

When the bat strikes the ball, the magnitude of the force exerted on the ball rises to a maximum value and then returns to zero

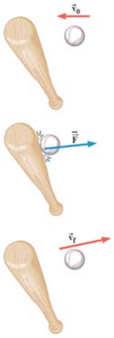


The collision time between a bat and a ball is very short, often less than a millisecond, but the force can be quite large.

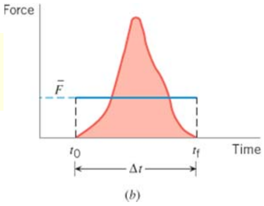
The time interval during which the force acts is Δt, and the magnitude of the average force is F.

DEFINITION OF IMPULSE

The impulse of a force is the product of the average force and the time interval during which the force acts:



$$\vec{\mathbf{J}} = \overline{\vec{\mathbf{F}}}\Delta t$$

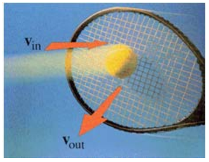


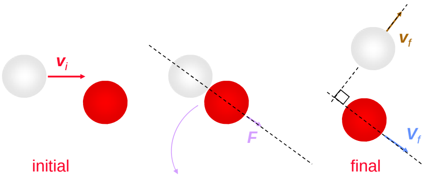
Impulse is a vector quantity and has the same direction as the average force.

SI unit: $\text{newton}\cdot\text{seconds}\ (\text{N}\cdot\text{s})$

Momentum transfer (collision) ‘timescales’

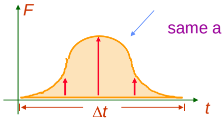
- Collisions typically involve interactions that happen quickly.
- During this brief time, the forces involved can be quite large





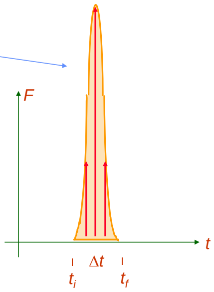
The balls are in contact for a very short time.

Force and Impulse



Δt big, F small

$$\vec{\mathbf{J}} = \overline{\vec{\mathbf{F}}}\Delta t$$



Δt small, F big

DEFINITION OF LINEAR MOMENTUM

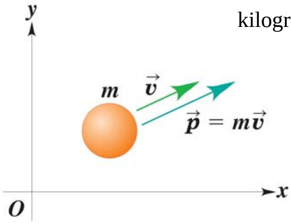
The linear momentum of an object is the product of the object's mass times its velocity:

$$\vec{\mathbf{p}} = m\vec{\mathbf{v}}$$

Momentum $\vec{p}$ is a vector quantity; a particle's momentum has the same direction as its velocity $\vec{v}$.

SI unit:

kilogram · meter/second (kg · m/s)



Linear Momentum

Momentum of a particle is defined as the product of its mass and velocity

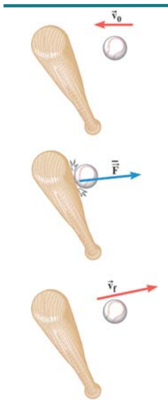
$$\vec{p} = m\,\vec{v}$$

- **Momentum components**
  - **$p_x = m\,v_x$ and $p_y = m\,v_y$**
  - **Applies to two-dimensional motion as well**

Momentum (magnitude) is related to kinetic energy

$$K = \frac{1}{2}mv^2 = \frac{p^2}{2m}$$

Relation between Impulse and Momentum (Newton 2nd)



$$\vec{\mathbf{a}} = \frac{\vec{\mathbf{v}}_f - \vec{\mathbf{v}}_o}{\Delta t}$$

$$\sum \vec{\mathbf{F}} = m\vec{\mathbf{a}}$$

$$\sum \vec{\mathbf{F}} = \frac{m\vec{\mathbf{v}}_f - m\vec{\mathbf{v}}_o}{\Delta t}$$

$$\left(\sum \vec{\mathbf{F}}\right)\Delta t = m\vec{\mathbf{v}}_f - m\vec{\mathbf{v}}_o$$

IMPULSE-MOMENTUM THEOREM

When a net force acts on an object, the impulse of this force is equal to the change in the momentum of the object

final momentum

initial momentum

impulse

$$\left(\sum \vec{\mathbf{F}}\right)\Delta t = m\vec{\mathbf{v}}_f - m\vec{\mathbf{v}}_o$$

$$\vec{J} = \Delta\vec{p} = \vec{F}\,\Delta t$$

Impulse is a vector quantity;
SI Unit: N s or kg m / s

impulse = change in momentum!

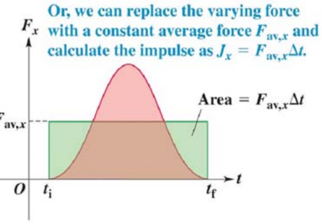
Impulse and average force

- **We can use the notion of impulse to define "average force", which is a useful concept.**

Define average force

such that (even if
is not constant),
impulse is given by

$\vec{F}_{av}$

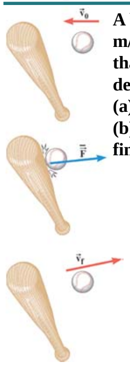


$$\vec{J} = \vec{F}_{av}\,(t_f - t_i) = \vec{F}_{av}\,\Delta t$$

or:

$$\vec{F}_{av} = \frac{\Delta\vec{p}}{\Delta t}$$

Example: A Well Hit Ball



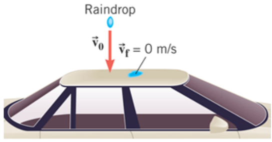
A baseball (m = 0.14 kg) has an initial velocity of v_o = - 38 m/s as it approaches a bat. The bat applies an average force that is much larger than the weight of the ball, and the ball departs from the bat with a final velocity of v_f = 58 m/s .
(a) Determine the impulse applied to the ball by the bat.
(b) Assuming that the time of contact is $\Delta t = 1.6 \times 10^{-3}$ s, find the average force exerted on the ball by the bat.

$$\begin{aligned}\vec{J} &= m\vec{\mathbf{v}}_f - m\vec{\mathbf{v}}_0 \\ &= \underbrace{(0.14\text{ kg})(+58\text{ m/s})}_{\text{Final momentum}} - \underbrace{(0.14\text{ kg})(-38\text{ m/s})}_{\text{Initial momentum}} \\ &= \boxed{+13.4\text{ kg}\cdot\text{m/s}}\end{aligned}$$

$$\begin{aligned}\vec{\overline{\mathbf{F}}} &= \frac{\vec{\mathbf{J}}}{\Delta t} = \frac{+13.4\text{ kg}\cdot\text{m/s}}{1.6\times 10^{-3}\text{ s}} \\ &= \boxed{+8400\text{ N}}\end{aligned}$$

Example 2: A Rainstorm

Rain comes down with a velocity of -15 m/s and hits the roof of a car. The mass of rain per second that strikes the roof of the car is 0.060 kg/s. Assuming that rain comes to rest upon striking the car, find the average force exerted by the rain on the roof.



$$\left(\sum \overline{\vec{\mathbf{F}}}\right)\Delta t = m\vec{\mathbf{v}}_{\mathbf{f}} - m\vec{\mathbf{v}}_{\mathbf{o}}$$

Neglecting the weight of the raindrops, the net force on a raindrop is simply the force on the raindrop due to the roof.

$$\overline{\vec{\mathbf{F}}}\ \Delta t = m\vec{\mathbf{v}}_{\mathbf{f}} - m\vec{\mathbf{v}}_{\mathbf{o}} \quad\Longrightarrow\quad \overline{\vec{\mathbf{F}}} = -\left(\frac{m}{\Delta t}\right)\vec{\mathbf{v}}_{\mathbf{o}}$$

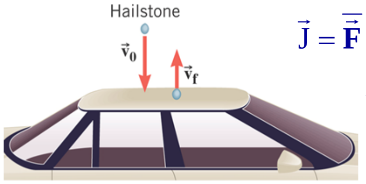
$$\overline{\vec{\mathbf{F}}} = -\left(0.060\,\text{kg/s}\right)\left(-15\,\text{m/s}\right) = +0.90\ \text{N (force on the raindrop)}$$

$F_{\text{on-the-roof}} = -0.90$ F (Newton's third law)

Conceptual Example: Hailstones versus Raindrops

Instead of rain, suppose hail is falling. Unlike rain, hail usually bounces off the roof of the car.

If hail fell instead of rain, would the force be smaller than, equal to, or greater than that calculated previously ?



$$\vec{J} = \overline{\vec{\mathbf{F}}}\ \Delta t = m\vec{\mathbf{v}}_{\mathbf{f}} - m\vec{\mathbf{v}}_{\mathbf{o}}$$

$$\overline{\vec{\mathbf{F}}} = \frac{m\vec{\mathbf{v}}_{\mathbf{f}} - m\vec{\mathbf{v}}_{\mathbf{o}}}{\Delta t}$$

For a raindrop, the change in velocity is from (downward) to zero.
For a hailstone, the change is from (downward) to (upward).

Thus hailstones have a larger $\overline{\vec{\mathbf{F}}}\ \Delta t$

Impulse applied to auto collisions

- The most important factor is the collision time or the time it takes the person to come to a rest
  - This will reduce the chance of dying in a car crash
- Ways to increase the time
  - Seat belts
  - Air bags

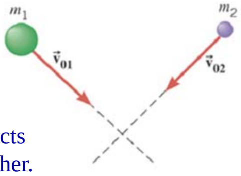
$$\vec{F}_{av} = \frac{\Delta\vec{p}}{\Delta t} = \frac{m\left(\vec{v}_f - \vec{v}_i\right)}{\Delta t}$$

➢ The air bag increases the time of the collision and absorbs some of the energy from the body

Conservation of Linear Momentum

WORK-ENERGY THEOREM ⇔ CONSERVATION OF ENERGY

Apply the impulse-momentum theorem to the midair collision between two objects…..



Internal forces – Forces that objects within the system exert on each other.

External forces – Forces exerted on objects by agents external to the system.

e.g. Weight=W

Conservation of Linear Momentum

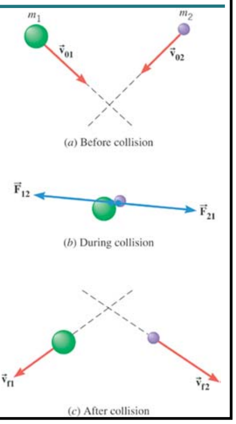
$$\left(\sum \overline{\vec{\mathbf{F}}}\right)\Delta t = m\vec{\mathbf{v}}_{\mathbf{f}} - m\vec{\mathbf{v}}_{\mathbf{o}}$$

OBJECT 1

$$\left(\vec{\mathbf{W}}_1 + \overline{\vec{\mathbf{F}}}_{12}\right)\Delta t = m_1\vec{\mathbf{v}}_{\mathbf{f}1} - m_1\vec{\mathbf{v}}_{\mathbf{o}1}$$

OBJECT 2

$$\left(\vec{\mathbf{W}}_2 + \overline{\vec{\mathbf{F}}}_{21}\right)\Delta t = m_2\vec{\mathbf{v}}_{\mathbf{f}2} - m_2\vec{\mathbf{v}}_{\mathbf{o}2}$$



(a) Before collision

(b) During collision

(c) After collision

Conservation of Linear Momentum

$$\left(\vec{\mathbf{W}}_1 + \overline{\vec{\mathbf{F}}}_{12}\right)\Delta t = m_1\vec{\mathbf{v}}_{\mathbf{f}1} - m_1\vec{\mathbf{v}}_{\mathbf{o}1}$$

Consider system: both objects involved

$$+$$

$$\left(\vec{\mathbf{W}}_2 + \overline{\vec{\mathbf{F}}}_{21}\right)\Delta t = m_2\vec{\mathbf{v}}_{\mathbf{f}2} - m_2\vec{\mathbf{v}}_{\mathbf{o}2}$$

$$\Downarrow$$

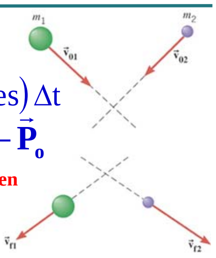
$$\left(\vec{\mathbf{W}}_1 + \vec{\mathbf{W}}_2 + \underset{\substack{\uparrow\\ \overline{\vec{\mathbf{F}}}_{12} = -\overline{\vec{\mathbf{F}}}_{21}}}{\overline{\vec{\mathbf{F}}}_{12}} + \overline{\vec{\mathbf{F}}}_{21}\right)\Delta t = \underset{\substack{\uparrow\\ \vec{\mathbf{P}}_{\mathbf{f}}}}{\left(m_1\vec{\mathbf{v}}_{\mathbf{f}1} + m_2\vec{\mathbf{v}}_{\mathbf{f}2}\right)} - \underset{\substack{\uparrow\\ \vec{\mathbf{P}}_{\mathbf{o}}}}{\left(m_1\vec{\mathbf{v}}_{\mathbf{o}1} + m_2\vec{\mathbf{v}}_{\mathbf{o}2}\right)}$$

The internal forces cancel out.

Principle of Conservation of Linear Momentum

$$\left(\vec{\mathbf{W}}_1 + \vec{\mathbf{W}}_2\right)\Delta t = \vec{\mathbf{P}}_\mathbf{f} - \vec{\mathbf{P}}_\mathbf{o}$$

$$\left(\text{sum of average external forces}\right)\Delta t = \vec{\mathbf{P}}_\mathbf{f} - \vec{\mathbf{P}}_\mathbf{o}$$



If the sum of the external forces is zero, then

$$0 = \vec{\mathbf{P}}_\mathbf{f} - \vec{\mathbf{P}}_\mathbf{o} \Longrightarrow \boxed{\vec{\mathbf{P}}_\mathbf{f} = \vec{\mathbf{P}}_\mathbf{o}}$$

CONSERVATION OF LINEAR MOMENTUM

The total linear momentum of an isolated system is constant (conserved). An isolated system is one for which the sum of the average external forces acting on the system is zero.

Definition of Total Momentum for a System of Particles

- **For a system of particles the total momentum *P* is the vector sum of the individual particle momenta:**

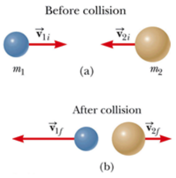
$$\vec{P} = \sum_{i=1}^{N} \vec{p}_i = \sum_{i=1}^{N} m_i \vec{v}_i$$

$$\vec{P} = \vec{p}_A + \vec{p}_B + \vec{p}_C + \ldots \quad = m_A\vec{v}_A + m_B\vec{v}_B + m_C\vec{v}_C + \ldots$$

components of total momentum $P_x = p_{A,x} + p_{B,x} + \ldots\ldots$

$P_y = p_{A,y} + p_{B.y} + \ldots\ldots$

$$\boxed{\Delta\vec{P} = \Delta(\vec{p}_A + \vec{p}_B + \vec{p}_C + \ldots) = \vec{F}_{net}\,\Delta t}$$



- **The momentum of each object will change**
- **The total momentum of the system remains constant if there are no external forces**

Example: Assembling a Freight Train

A freight train is being assembled in a switching yard. Car 1 has a mass of m_1 = 65 × 10^3 kg and moves at a velocity of v_{01} = +0.80 m/s. Car 2, with a mass of m_2 = 92 × 10^3 kg and a velocity of v_{02} = +1.3 m/s, overtakes car 1 and couples to it. Neglecting friction, find the common velocity v_f of the cars after they become coupled.

(*a*) Before coupling

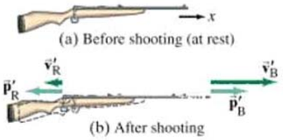
(*b*) After coupling

$$\underbrace{m_1\mathbf{v}_{01} + m_2\mathbf{v}_{02}}_{\substack{\text{Total momentum}\\ \text{before collision}}} = \underbrace{(m_1 + m_2)\mathbf{v}_\mathbf{f}}_{\substack{\text{Total momentum}\\ \text{after collision}}}$$

$$\mathbf{v}_\mathbf{f} = \frac{m_1 v_1 + m_2 v_2}{m_1 + m_2}$$

$$= \frac{\left(65\times10^3\text{ kg}\right)(0.80\text{ m/s}) + \left(92\times10^3\text{ kg}\right)(1.3\text{ m/s})}{\left(65\times10^3\text{ kg} + 92\times10^3\text{ kg}\right)} = \boxed{+1.1\text{ m/s}}$$

Example: Recoil of a rifle



(a) Before shooting (at rest)

(b) After shooting

A marksman holds a 3.00 kg rifle loosely, allowing it to recoil freely when fired, and fires a bullet of mass 5.00 g horizontally with a speed v_B = 300 m/s. What is the recoil speed of the rifle ?

$$\vec{P}_T = \vec{P}_R + \vec{P}_B$$

$$Before \qquad \vec{P}_T = 0$$

$$After \qquad \vec{P}_R + \vec{P}_B = 0$$

$$m_B v_B + m_R v_R = 0$$

$$v_R = -\frac{m_B}{m_R}\cdot v_B = -\frac{0.005}{3.00}\cdot 300 = -0.50\,m/s.$$

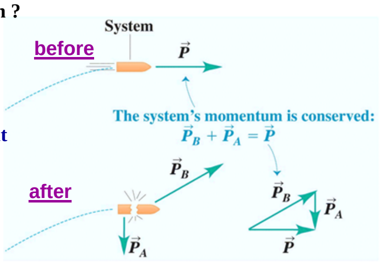
Concept Test: Exploding Projectile

A model rocket travels as a projectile in a parabolic path after its first stage burns out. At the top of its trajectory, where its velocity points horizontally to the right, a small explosion separates it into two sections with equal masses. One section falls straight down, with no horizontal motion. What is the direction of the other part just after the explosion ?

A. Up and to the left

B. Straight up

[✓ C. Up and to the right]



The system's momentum is conserved:
$\vec{P}_B + \vec{P}_A = \vec{P}$

Example: Momentum Conservation

A box with mass *m* = 6.0 kg slides with speed *v* = 4.0 m/s across a frictionless floor in the positive direction of an *x* axis. It suddenly explodes into two pieces. One piece, with mass m_1 = 2.0 kg, moves in the positive x-direction with speed v_1 = 8.0 m/s.

What is the velocity of the second piece, with mass m_2 = 4.0 kg ?

$$P_i = P_f$$

or $$mv = m_1v_1 + m_2v_2.$$

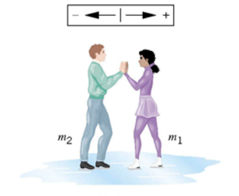
Inserting known data, we find

$$(6.0\text{ kg})(4.0\text{ m/s}) = (2.0\text{ kg})(8.0\text{ m/s}) + (4.0\text{ kg})v_2$$

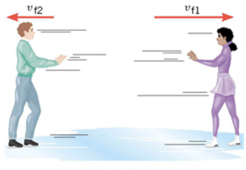
and thus $$v_2 = 2.0\text{ m/s}.$$

Since the result is positive, the second piece moves in the positive direction of the *x* axis.

Example: Conservation of Linear Momentum - Ice Skaters



(a) Before



(b) After

Starting from rest, two skaters push off against each other on ice where friction is negligible. One is a 54-kg woman and one is a 88-kg man. The woman moves away with a speed of +2.5 m/s. Find the recoil velocity of the man.

$$\vec{\mathbf{P}}_{\mathbf{f}} = \vec{\mathbf{P}}_{\mathbf{o}}$$

$$m_1 v_{f1} + m_2 v_{f2} = 0$$

$$v_{f2} = -\frac{m_1 v_{f1}}{m_2}$$

$$v_{f2} = -\frac{(54\,kg)\left(+2.5\frac{m}{s}\right)}{88\,kg} = -1.5\frac{m}{s}$$

Concept Test: Conservation of Momentum

A boy stands at one end of a floating raft that is stationary relative to the shore. He then walks to the opposite end, towards the shore. Does the raft move (assume no friction)?

1. No, it will not move at all

[✓ 2.] Yes, it will move away from the shore ✓

3. Yes, it will move towards the shore

Note: Since momentum is conserved in the boy-raft system and neither was moving at first, the raft must move in the direction opposite to the boy's.

In Collissions Total Momentum is Conserved

In collisions, we assume that external forces either sum to zero, or are small enough to be ignored. Hence, momentum is conserved in all collisions.

- A collision may be the result of physical contact between two objects
- "Contact" may also arise from the electrostatic interactions of the electrons in the surface atoms of the bodies

- Mathematically (for two objects):

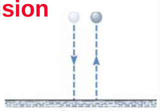
$$\boxed{m_1 \vec{v}_{1i} + m_2 \vec{v}_{2i} = m_1 \vec{v}_{1f} + m_2 \vec{v}_{2f}}$$

  - Momentum is conserved for the *system* of objects
  - The system includes all the objects interacting with each other
  - Assumes only internal forces are acting during the collision
  - Can be generalized to any number of objects

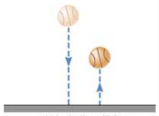
Types of Collisions

- **Momentum is conserved in any collision**
- **Elastic collisions**
  - **both momentum and kinetic energy are conserved**
- **Inelastic collisions**
  - **Kinetic energy is not conserved**
    - **Some of the kinetic energy is converted into other types of energy such as heat, sound, work to permanently deform an object**
  - **Perfectly inelastic collisions occur when the objects stick together**
    - **Not all of the KE is necessarily lost**

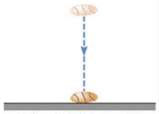
Most collisions fall between elastic and perfectly inelastic collisions



(a) Elastic collision



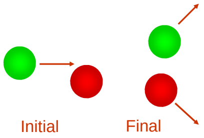
(b) Inelastic collision



(c) Completely inelastic collision

Elastic Collisions

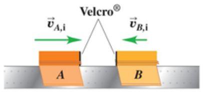
- Elastic means that kinetic energy is conserved as well as momentum.
- This gives us more constraints
  - We can solve more complicated problems!!
  - Billiards (2-D collision)
  - The colliding objects have separate motions after the collision as well as before.



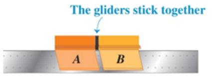
- First: simpler 1-D problem

Inelastic and Elastic Collisions

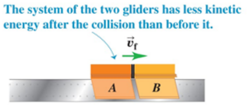
A completely inelastic collision



(a) Before collision

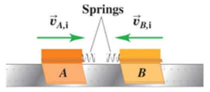


(b) Completely inelastic collision

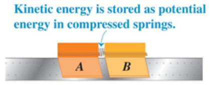


(c) After collision

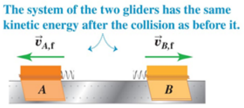
An elastic collision



(a) Before collision



(b) Elastic collision



(c) After collision

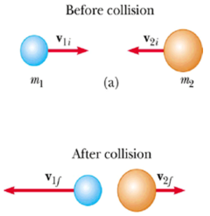
Applying the Principle of Conservation of Momentum

1. **Decide which objects are included in the system.**
2. **Relative to the system, identify the internal and external forces.**
3. **Verify that the system is isolated.**
4. **Set the final momentum of the system equal to its initial momentum.**

Remember that momentum is a vector.

Problem Solving for One-Dimensional Collisions

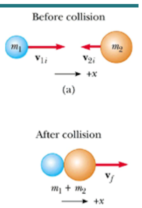
- **Set up a coordinate axis and define the velocities with respect to this axis**
  - **It is convenient to make your axis coincide with one of the initial velocities**
- **In your sketch, draw all the velocity vectors with labels including all the given information**
- **Draw “before” and “after” sketches**
- **Label each object**
  - **include the direction of velocity**
  - **keep track of subscripts**



Problem Solving for One-Dimensional Collisions

Sketch for perfectly inelastic colllision

- **The objects stick together**
- **Include all the velocity directions**
- **The “after” collision combines the masses**



- **Write the expressions for the momentum of each object before and after the collision**
  - **Remember to include the appropriate signs**
- **Write an expression for the total momentum before and after the collision --- momentum of the system is conserved**
- **If the collision is inelastic, solve the momentum equation for the unknown --- Remember, KE is not conserved**
- **If the collision is elastic, you can use the KE equation to solve for two unknowns**

Perfectly Inelastic Collisions

- Suppose, for example, v_{2i}=0. Conservation of momentum becomes

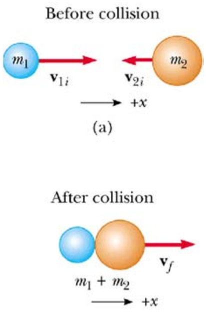
$$m_1 v_{1i} + m_2 v_{2i} = (m_1 + m_2) v_f$$

$$m_1 v_{1i} + 0 = (m_1 + m_2) v_f$$

E.g., if $m_1 = 1000$ kg, $m_2 = 1500$ kg :

$$(1000\text{kg})(50\,\text{m/s}) + 0 = (2500\text{kg})v_f \; ,$$

$$v_f = \frac{5\times10^4\,\text{kg}\cdot\text{m/s}}{2.5\times10^3\,\text{kg}} = \underline{20\,\text{m/s}}.$$



Perfectly Inelastic Collisions

- What amount of KE lost during collision?

$$KE_{\text{before}} = \frac{1}{2} m_1 v_{1i}^2 + \frac{1}{2} m_2 v_{2i}^2$$

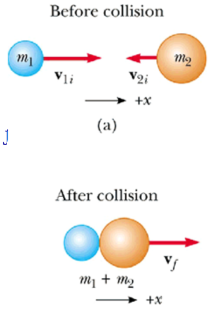
$$= \frac{1}{2}(1000\text{ kg})(50\,\text{m/s})^2 = 1.25\times10^6\,\text{J}$$

$$KE_{\text{after}} = \frac{1}{2}(m_1 + m_2) v_f^2$$

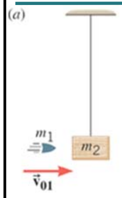
$$= \frac{1}{2}(2500\text{ kg})(20\,\text{m/s})^2 = 0.50\times10^6\,\text{J}$$

$$\Delta KE_{lost} = \underline{0.75\times10^6\,J}$$

lost in heat and sound …



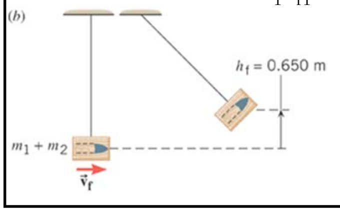
Example: A Ballistic Pendulum



The mass of the block of wood is 2.50-kg and the mass of the bullet is 0.0100-kg. The block swings to a maximum height of 0.650 m above the initial position. Find the initial speed of the bullet.

Apply conservation of momentum to the collision:

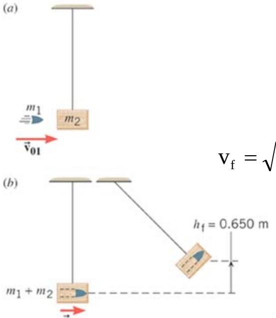
$$m_1 v_{f1} + m_2 v_{f2} = m_1 v_{o1} + m_2 v_{o2}$$



$$\left(m_1 + m_2\right) v_f = m_1 v_{o1}$$

$$v_{o1} = \frac{\left(m_1 + m_2\right) v_f}{m_1}$$

Applying conservation of energy to the swinging motion:



$$mgh = \tfrac{1}{2}mv^2$$

$$(m_1 + m_2)gh_f = \tfrac{1}{2}(m_1 + m_2)v_f^2$$

$$g\,h_f = \tfrac{1}{2}v_f^2$$

$$v_f = \sqrt{2gh_f} = \sqrt{2(9.80\,m/s^2)(0.650\,m)}$$

$$v_{o1} = \frac{(m_1 + m_2)v_f}{m_1}$$

$$v_{o1} = \left(\frac{0.0100\,kg + 2.50\,kg}{0.0100\,kg}\right)\sqrt{2(9.80\,m/s^2)(0.650\,m)} = +896\,m/s$$

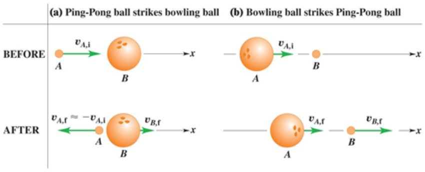
Elastic Collisions

- **Both momentum and kinetic energy are conserved**
- **Typically have two unknowns**

- Solve the equations simultaneously

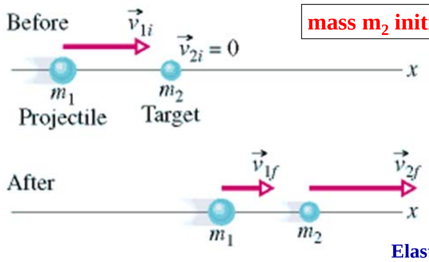
$$m_1v_{1i} + m_2v_{2i} = m_1v_{1f} + m_2v_{2f}$$

$$\frac{1}{2}m_1v_{1i}^2 + \frac{1}{2}m_2v_{2i}^2 = \frac{1}{2}m_1v_{1f}^2 + \frac{1}{2}m_2v_{2f}^2$$



- Incoming and outgoing velocities are very mass dependant

mass m_2 initially at rest



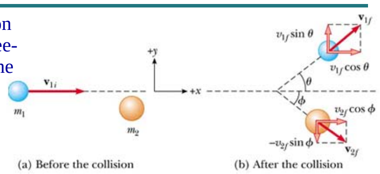
Elastic Collision

$$m_1v_{1i} + m_2v_{2i} = m_1v_{1f} + m_2v_{2f} \qquad (1)$$

$$\frac{1}{2}m_1v_{1i}^2 + \frac{1}{2}m_2v_{2i}^2 = \frac{1}{2}m_1v_{1f}^2 + \frac{1}{2}m_2v_{2f}^2 \qquad (2)$$

Two-dimensional Collisions

- For a general collision of two objects in three-dimensional space, the conservation of momentum principle



(a) Before the collision (b) After the collision

… implies that the *total momentum of the system in <u>each</u> <u>direction</u> is conserved*

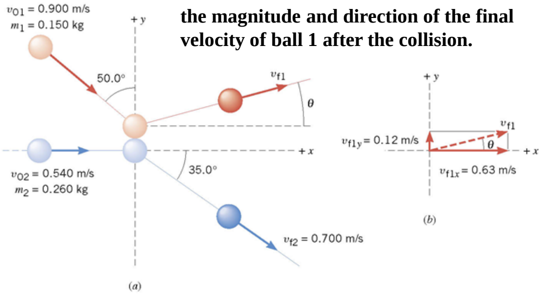
$$\boxed{m_1\vec{v}_{1i} + m_2\vec{v}_{2i} = m_1\vec{v}_{1f} + m_2\vec{v}_{2f}}$$

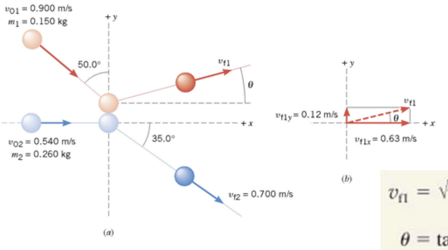
- – Use subscripts for identifying the object, initial and final, and components

$$\boxed{\begin{aligned} &m_1v_{1ix} + m_2v_{2ix} = m_1v_{1fx} + m_2v_{2fx} \text{ and} \\ &m_1v_{1iy} + m_2v_{2iy} = m_1v_{1fy} + m_2v_{2fy} \end{aligned}}$$

A Collision in Two Dimensions

Use momentum conservation to determine the magnitude and direction of the final velocity of ball 1 after the collision.



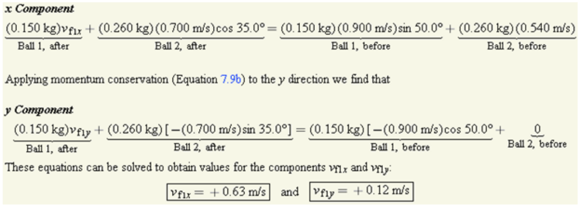


$$v_{f1} = \sqrt{v_{f1x}^2 + v_{f1y}^2}$$

$$\theta = \tan^{-1}\left(\frac{v_{f1y}}{v_{f1x}}\right)$$

$$m_1v_{f1x} + m_2v_{f2x} = m_1v_{o1x} + m_2v_{o2x}$$

$$m_1v_{f1y} + m_2v_{f2y} = m_1v_{o1y} + m_2v_{o2y}$$



x Component

$$\underbrace{(0.150\text{ kg})v_{f1x}}_{\text{Ball 1, after}} + \underbrace{(0.260\text{ kg})(0.700\text{ m/s})\cos 35.0°}_{\text{Ball 2, after}} = \underbrace{(0.150\text{ kg})(0.900\text{ m/s})\sin 50.0°}_{\text{Ball 1, before}} + \underbrace{(0.260\text{ kg})(0.540\text{ m/s})}_{\text{Ball 2, before}}$$

Applying momentum conservation (Equation 7.9b) to the *y* direction we find that

y Component

$$\underbrace{(0.150\text{ kg})v_{f1y}}_{\text{Ball 1, after}} + \underbrace{(0.260\text{ kg})[-(0.700\text{ m/s})\sin 35.0°]}_{\text{Ball 2, after}} = \underbrace{(0.150\text{ kg})[-(0.900\text{ m/s})\cos 50.0°}_{\text{Ball 1, before}} + \underbrace{0}_{\text{Ball 2, before}}$$

These equations can be solved to obtain values for the components v_{f1x} and v_{f1y}:

$$\boxed{v_{f1x} = +0.63\text{ m/s}} \quad \text{and} \quad \boxed{v_{f1y} = +0.12\text{ m/s}}$$

$$v_{f1} = \sqrt{v_{f1x}^2 + v_{f1y}^2} \qquad (1a)$$

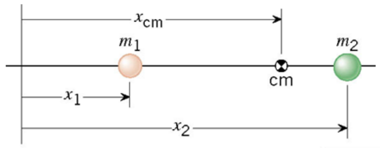
$$\boxed{v_{f1x} = +0.63\text{ m/s}} \quad \boxed{v_{f1y} = +0.12\text{ m/s}}$$

$$\theta = \tan^{-1}\left(\frac{v_{f1y}}{v_{f1x}}\right) \qquad (1b)$$

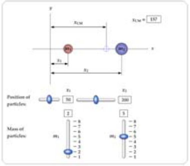
$$\boxed{v_{f1x} = +0.63\text{ m/s}} \quad \boxed{v_{f1y} = +0.12\text{ m/s}}$$

Center of Mass

The center of mass is a point that represents the average location for the total mass of a system.



$$x_{cm} = \frac{m_1 x_1 + m_2 x_2}{m_1 + m_2}$$



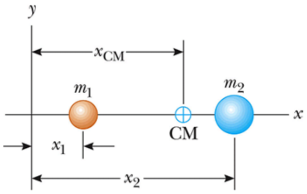
Center of Mass Coordinates

- The coordinates of the center of mass are

$$x_{CM} = \frac{\sum_i m_i x_i}{\sum_i m_i}$$

$$y_{CM} = \frac{\sum_i m_i y_i}{\sum_i m_i}$$

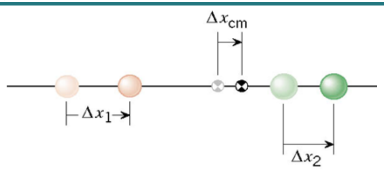
$\sum_i m_i$ = M is the total mass of the system



Two masses on x-axis

$$x_{CM} = \frac{m_1 x_1 + m_2 x_2}{m_1 + m_2}$$

Velocity of Center of Mass



$$\Delta x_{cm} = \frac{m_1 \Delta x_1 + m_2 \Delta x_2}{m_1 + m_2} \Rightarrow \frac{\Delta x_{cm}}{\Delta t} = \frac{m_1 \frac{\Delta x_1}{\Delta t} + m_2 \frac{\Delta x_2}{\Delta t}}{m_1 + m_2}$$

$$v_{cm} = \frac{m_1 v_1 + m_2 v_2}{m_1 + m_2} \Rightarrow \boxed{p_{cm} = M\ v_{cm} = p_1 + p_2}$$

In an <u>isolated</u> system, the total linear momentum does not change, therefore the velocity of the center of mass does not change.

Example: Center of Mass Motion

v_{cm} = const. In isolated system

BEFORE

$$v_{cm} = \frac{m_1 v_1 + m_2 v_2}{m_1 + m_2} = 0$$

(a) Before

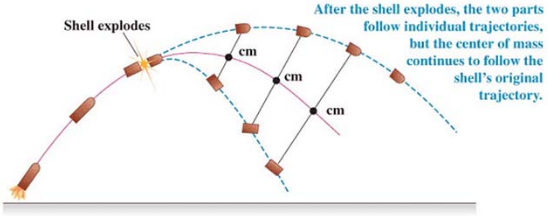
(b) After

AFTER

$$v_{cm} = \frac{(88\text{ kg})(-1.5\text{ m/s}) + (54\text{ kg})(+2.5\text{ m/s})}{88\text{ kg} + 54\text{ kg}} = 0.002 \approx 0$$

Motion of the Center of Mass

- The system will move as if an external force were applied to a single particle of mass *M* located at the center of mass



After the shell explodes, the two parts follow individual trajectories, but the center of mass continues to follow the shell's original trajectory.